

SPRING FORCE

FRICITION

GRAVITY

SET POINT

DISPLACEMENT (X)

VELOCITY $\frac{dx}{dt} = \dot{x}$

ACCELERATION $\frac{d^2x}{dt^2} = \ddot{x} = \frac{dv}{dt}$

ANY ATTEMPT TO CHANGE THE SET POINT WILL RESULT IN OSCILLATION

UNLESS THE FRICTION IS ENOUGH TO STOP IT

MASS M

SPRING CONSTANT K

FRICTION COEFF. (DAMPER) b

INPUT FORCING FUNCTION $F(t)$

$$M\ddot{x} + b\dot{x} + Kx = F(t)$$

$$M\ddot{x} + b\dot{x} + kx = F(\tau)$$

\uparrow
 SECOND DERIVATIVE

LAPLACE TRANSFORMATION

$X(\tau) \rightarrow X(s)$
 $\dot{x}(\tau) \rightarrow sX(s)$
 $\ddot{x}(\tau) \rightarrow s^2 X(s)$

$M s^2 X(s) + b s X(s) + k X(s) = F(s)$

LINEAR SECOND ORDER DIFFERENTIAL EQUATION WITH CONSTANT COEFFICIENTS

\uparrow
 THIS IS WHAT ALLOWS US TO USE LAPLACE

$$Ms^2x(s) + bsx(s) + kx(s) = F(s)$$

IF THE INITIAL CONDITIONS ARE ZERO

$$X(s) [Ms^2 + bs + k] = F(s)$$

$$Ri + \left[\frac{1}{C} \int_0^{\tau} i d\tau \right] + L \frac{di}{d\tau} = V \text{ (KVL)}$$

$$R \frac{di}{d\tau} + \frac{i}{C} + L \frac{d^2i}{d\tau^2} = \frac{dV}{d\tau}$$

$$L \frac{d^2i}{d\tau^2} + R \frac{di}{d\tau} + \frac{i}{C} = 0$$

$$L s^2 i(s) + R s i(s) + \frac{1}{C} i(s) = 0$$

SOLUTION FOR WHAT HAPPENS TO $i(s)$

$$L s^2 + R s + \frac{1}{C} = 0$$

$V = V$ (AT THE END)
 $i = 0$

THAT SOLUTION FOR $i(s)$
HAS 3 POSSIBLE OUTCOMES

1. OSCILLATORY (UNDER DAMPED)
2. CRITICALLY DAMPED
3. OVER DAMPED

