Solutions Class Test 2
Nov 2020

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1. (a) $\Omega = \frac{2\pi f}{f_s}$ 1 $\quad \Omega_0 = \frac{2\pi\, 0.75}{25} = 0.19$ 1

$$x[n] = \sin(n\Omega_0) = \sin(0.19n)$$ 2

(b) $H(\Omega) = \frac{Y(\Omega)}{X(\Omega)} = 3 - 2e^{-j\Omega} + 4e^{-j2\Omega}$ 4

(c) $h[n] = 3\delta[n] - 2\delta[n-1] + 4\delta[n-2]$

$$H(\Omega) = \sum_{n=-\infty}^{\infty} h[n] e^{-jn\Omega}$$ DTFT 1

$$H(\Omega) = h[0]e^{0} + h[1]e^{-j\Omega} + h[2]e^{-j2\Omega} + h[3]e^{-j3\Omega} + \ldots\ldots$$ 1

$h[0] = 3,\ h[1] = -2,\ h[2] = 4,$

$h[n \geq 3] = 0$

$$H(\Omega) = 3 - 2e^{-j\Omega} + 4e^{-j2\Omega}$$ 1

1. (d) $H(\Omega) = 3 - 2e^{-j\Omega} + 4e^{-j2\Omega}$

$H(\Omega_0) = 3 - 2e^{-j0.1885} + 4e^{-j0.377}$ 2

$H(\Omega_0) = (3\angle 0) - (2\angle -0.1885) + (4\angle -0.377)$

$|H(\Omega_0)| = 4.88$ 2 $\quad \Phi(\Omega_0) = -13^0$ 2

(e) $H(0) = 3 - 2 + 4 = 5$ 4

(f) $y[n] = 5\sin(n\Omega)$ 2

SINCE $\Omega = 0$, $\sin(0) = 0$ 1

THEREFORE $x[n]$ AND $y[n]$ BOTH ARE ZERO FOR ALL VALUES OF n 1

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2. (a) $$H(z) = \frac{1}{0.8 + 0.23z^{-1} + 0.15z^{-2}}$$

MULTIPLY BY z^2

$$H(z) = \frac{z^2}{0.8z^2 + 0.23z + 0.15} \quad 1$$

DIVIDE BY 0.8

$$H(z) = \frac{1.25z^2}{z^2 + 0.2875z + 0.1875} \quad 1$$

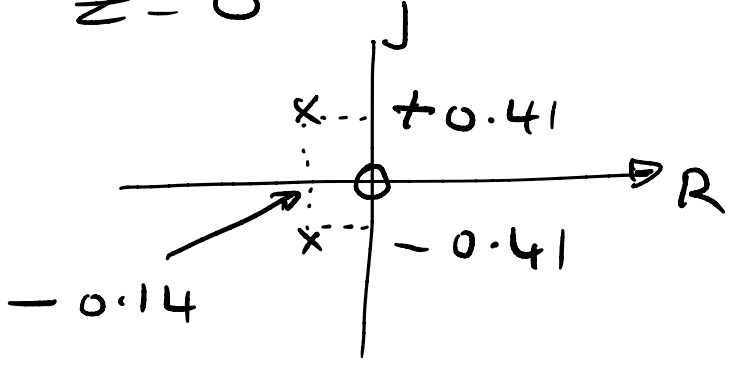
POLES

$$z = \frac{-0.2875 \pm \sqrt{(0.2875)^2 - 4(0.1875)}}{2} \quad 1$$

$$= -0.14 \pm j\frac{\sqrt{0.66734375}}{2}$$

$$= -0.14 \pm j0.41 \quad 1$$

ZERO AT $z = 0$



2

$$|z| = \sqrt{(-0.14)^2 + (0.41)^2}$$

$$= 0.43$$ FILTER IS STABLE

2

DIFF EQN.

$$0.8\,y[n] + 0.23\,y[n-1] + 0.15\,y[n-2] = x[n]$$

1

$$y[n] = -0.29\,y[n-1] - 0.19\,y[n-2] + 1.25\,x[n]$$

$$h[n] = -0.29\,h[n-1] - 0.19\,h[n-2] + 1.25\,\delta[n]$$

1

(b) $$y[n] = -0.6\,y[n-1] - 0.13\,y[n-2] + x[n-1]$$

$$H(z) = \frac{z^{-1}}{1 + 0.6z^{-1} + 0.13z^{-2}}$$

2

$$H(z) = \frac{z}{z^2 + 0.6z + 0.13}$$

2

POLES

$$\frac{-0.6 \pm \sqrt{(0.6)^2 - 4(0.13)}}{2}$$

2

$$z = -0.3 \pm j\,0.2$$

1

THERE IS ZERO AT $z = 0$

$$|z| = \sqrt{(-0.3)^2 + (0.2)^2} = 0.36$$

2

$$s[n] = -0.6\,s[n-1] - 0.13\,s[n-2] + u[n-1]$$

1

$$y_{ss} + 0.6\,y_{ss} + 0.13\,y_{ss} = 1$$

1

$$y_{ss} = \frac{1}{1 + 0.6 + 0.13} = 0.58$$

1