UNIVERSITY OF THE WEST INDIES
CAVE HILL CAMPUS

Department of Computer Science, Mathematics & Physics

ELET3230 - Introduction to Digital Signal Processing

Assignment 2 **Due: Saturday, December 5, 2020**

1. (a) Show that the power of the periodic signal $s(t) = Acos(2\pi ft)$, given by $P = \frac{1}{T}\int_0^T s^2(t)dt$, is equal to $\frac{A^2}{2}$ using the identity $2cos^2\theta = 1 + cos(2\theta)$. Does the power of this cosine signal depend on it's frequency ? [4]

   (b) Express $s(t) = Acos(2\pi ft)$ as the addition of two phasors and use Parseval's theorem $\sum_{n=0}^{N-1}\left(\frac{|X_k|}{N}\right)^2$ to verify your answer to part(a). [3]

   (c) Given that the average value of a function $f(x)$ over the interval a to b is $\frac{1}{b-a}\int_a^b f(x)dx$, relate this to the expression $P = \frac{1}{T}\int_0^T s^2(t)dt$ to derive an expression for the power of a signal using the samples $x[n] = s\left(\frac{n}{f_s}\right)$, where f_s is the sampling frequency. [4]

   (d) Express the signal $s(t) = 1 + 2cos^2(2\pi ft)$ in terms of phasors and plot the corresponding double-sided amplitude spectrum. Use your spectrum to determine the power of this signal. [4]

2. Consider a filter with the difference equation

   $$y[n] = x[n] - x[n-1]$$

   (a) Determine the frequency response of this filter by using the equation $H(\Omega) = \frac{\sum_{k=0}^{M} b_k exp(-jk\Omega)}{\sum_{k=0}^{N} a_k exp(-jk\Omega)}$ where a_k and b_k are the difference equation coefficients. [4]

   (b) Prove that this filter removes the D.C. component of any signal passing through it. [4]

   (c) Determine the impulse response $h[n]$ and verify your answer to part (a) by taking DTFT of $h[n]$ using the equation $H(\Omega) = \sum_{n=-\infty}^{\infty} h[n]e^{-jn\Omega}$ [4]

   (d) For $x[n] = (0.8)^n$, the z-transform is $X(z) = \frac{z}{z-0.8}$ Determine the output $Y(z)$ for this filter [4]

   (e) Suppose we cascade two of these filters, show what effect this has on the poles and zeros of the transfer function. [4]

3. Plot the poles and zeroes of the digital filter

$$y[n] = -0.4y[n-1] + 0.5y[n-2] + 0.3x[n-1]$$ [7]

Check for filter stability [2]

Find the steady state output for the step response [2]